Int. J. Heat Mass Transfer. Vol. 27, No. 12, pp. 2253–2257, 1984
Printed in Great Britain

0017-9310/84 \$3.00 + 0.00

Comparisons between experiment and theory for dropwise condensation of mercury

J. NIKNEJAD and J. W. ROSE
Department of Mechanical Engineering, Queen Mary College, University of London, Mile End Road, London E1 4NS, U.K.

(*Received* 19 *October* 1983 *and in revised form* 16 *February* 1984)

Abstract—It is shown that convenient approximations, used in the theory of dropwise condensation to evaluate the integral giving the heat flux, lead to significant errors for the case of mercury. When these approximations are not adopted, and the integral is found numerically, agreement between theory and experiment is markedly improved.

INTRODUCTION

EXCELLENT agreement has been found between experiment and theory [1] for dropwise condensation of steam (see for instance ref. [2]). Recently similar close agreement has been found for the case of ethylene glycol [3]. For mercury, however, comparisons can only be described as fair [4]. In the case of mercury, theory [1, 5] indicates heat-transfer coefficients of the correct order of magnitude, but the experimental data (Q–ΔT relationship) shows curvature of opposite sign to that given by theory and found for steam and ethylene glycol. Moreover, to obtain closer agreement between theory and experiment for mercury, it is found necessary to assign to K_1 (a constant in the theory [1]) a value lower than expected. In the present note it is shown that these discrepancies are due to certain approximations which were made in the original theory to avoid using numerical methods and to obtain a relatively compact expression for the Q–ΔT relationship. It has been found that these approximations lead to much larger errors for mercury than for steam and ethylene glycol.

OUTLINE OF THEORY [1, 5]

The dropwise condensation process is modelled as a steady distribution of non-growing, non-coalescing drops. Equations for the heat-transfer rate through drops of given size and for the distribution of drop sizes are used to evaluate, by integration, the mean heat flux for the surface. The equation for the mean heat flux is

$$Q = \int_{\check{r}}^{\hat{r}} Q_\mathrm{b} \frac{r^{(1-n)/n}}{n\hat{r}^{1/n}}\,\mathrm{d}r \tag{1}$$

where Q_b, the heat flux through the base of a drop of radius r, is given, when zero thermal resistance is attributed to the promoter layer,* by

$$\frac{\Delta T}{T_\mathrm{v}} = \frac{2\sigma v_\mathrm{f} \sin\beta}{r h_\mathrm{fg}} + \frac{K_1 r}{k_\mathrm{f} T_\mathrm{v}} Q_\mathrm{b} + \frac{T_\mathrm{v} - T_\mathrm{s}}{T_\mathrm{v}}. \tag{2}$$

K_1 is a geometric factor for single drops and n is a parameter in the drop size distribution. For steam $n = 3$ and $K_1 = 2/3$ [1, 2]. T_s is the temperature of the liquid surface for which the corresponding saturation pressure P_s is given, from interphase mass-transfer theory (see ref. [1]), by

$$P_\mathrm{v} - P_\mathrm{s} = K_2(Q_\mathrm{b}/h_\mathrm{fg})(\{\gamma+1\}/\{\gamma-1\}) \times (RT_\mathrm{v}/2\pi)^{1/2} \tag{3}$$

where K_2 is the ratio of the base area of a drop to the area of its curved surface. Water drops approximate closely to hemispheres when $K_2 = 1/2$. In order to evaluate the integral in equation (1), it is necessary, for given values of T_v and ΔT, to solve equations (2) and (3) (noting that P_s and P_v are related through the saturation condition to T_s and T_v) to give Q_b for given values of r and to integrate numerically. The limits of integration are given [1, 5] by

$$\check{r} = 2\sigma v_\mathrm{f} T_\mathrm{v} \sin\beta / h_\mathrm{fg}\Delta T \tag{4}$$

and

$$\hat{r} = K_3(\sigma/g\rho_\mathrm{fg})^{1/2} \tag{5}$$

where K_3 is a dimensionless constant which was found to have a value of 0.4 both for water [1] and ethylene glycol [3].

If however, as in refs. [1, 5], the saturation curve is approximated, for small $T_\mathrm{v} - T_\mathrm{s}$, by a straight line (the tangent at P_v, T_v) we obtain an estimate for the interface temperature drop

$$T_\mathrm{v} - T_\mathrm{s} \approx \Delta T_\mathrm{e} = T_\mathrm{v} v_\mathrm{g}(P_\mathrm{v} - P_\mathrm{s})/h_\mathrm{fg} \tag{6}$$

when $v_\mathrm{g} > v_\mathrm{f}$. Taking $P_\mathrm{v} - P_\mathrm{s}$ from equation (3) and substituting for $T_\mathrm{v} - T_\mathrm{s}$ from equation (6) in equation (2), an expression for Q_b may be obtained. When this is used in equation (1) with $\check{r}$ and $\hat{r}$ from equations (4) and (5) we obtain

$$Q = \int_{\check{r} = 2\sigma v_\mathrm{f} T_\mathrm{v} \sin\beta/h_\mathrm{fg}\Delta T}^{\hat{r} = K_3(\sigma/g\rho_\mathrm{fg})^{1/2}} \left\{ \frac{\dfrac{\Delta T}{T_\mathrm{v}} - \dfrac{2\sigma}{r}\dfrac{v_\mathrm{f}}{h_\mathrm{fg}}\sin\beta}{\dfrac{K_1 r}{k_\mathrm{f} T_\mathrm{v}} + \dfrac{K_2 v_\mathrm{g}}{h_\mathrm{fg}^2}\left(\dfrac{\gamma+1}{\gamma-1}\right)\left(\dfrac{RT_\mathrm{v}}{2\pi}\right)^{1/2}} \right\} \times \left(\frac{g\rho_\mathrm{fg}}{K_3^2\sigma}\right)^{1/2n} \frac{r^{(1-n)/n}}{n}\,\mathrm{d}r. \tag{7}$$

---

* There is no promoter in the case of dropwise condensation of mercury on stainless steel.

NOMENCLATURE

g	specific force of gravity
h_{fg}	specific enthalpy of evaporation
K_1, K_2, K_3	constants in dropwise condensation theory
k_f	thermal conductivity of condensate
n	constant in dropwise condensation theory
P	pressure
P_s	$P_{sat}(T_s)$
$P_{sat}(T)$	saturation pressure at temperature T
P_v	pressure of vapour
Q	heat flux
Q_b	heat flux through base of drop
q	$\left\{\frac{K_2^2}{4\pi K_1}\left(\frac{\gamma+1}{\gamma-1}\right)^2\frac{RT_v^2k_fv_g^2}{\sigma h_{fg}^3v_f\sin\beta}\right\}Q$
R	specific ideal gas constant of condensing fluid
r	base radius of drop
$\check{r}$	base radius of smallest drop
$\hat{r}$	effective largest drop base radius
T	temperature
T_0	surface temperature
T_s	temperature of liquid side of vapour–condensate interface

T_v	temperature of vapour
ΔT	$T_v - T_0$
ΔT_e	estimate of $T_v - T_s$, see equation (6) and Fig. 4
v_f	specific volume of condensate
v_g	specific volume of vapour.

Greek symbols

α	heat-transfer coefficient, $Q/\Delta T$
β	contact angle
γ	ratio of principal specific heat capacities
θ	$\left\{\frac{1}{\sqrt{(8\pi)}}\frac{K_2}{K_1}\frac{(\gamma+1)}{(\gamma-1)}\frac{(RT_v)^{1/2}k_fv_g}{\sigma h_{fg}v_f\sin\beta}\right\}\Delta T$
θ_0	$\frac{1}{\sqrt{(2\pi)}}\frac{K_2}{K_1K_3}\frac{(\gamma+1)}{(\gamma-1)}\left(\frac{RT_vg\rho_{fg}}{\sigma}\right)^{1/2} \times \frac{k_fT_vv_g}{h_{fg}^2}$
ρ_{fg}	$v_f^{-1} - v_g^{-1}$
σ	surface tension.

Equation (7) may be integrated and the result expressed as

$$q\theta_0^{-1/n} = F_n(\theta) - \{(1+\theta)I_n(\theta_0^{1/n}) - \theta_0^{1-1/n}/(n-1)\} \quad (8)$$

where q and θ are dimensionless heat flux and temperature difference, respectively, and θ_0 involves the constants K_1, K_2 and K_3 and fluid properties. Expressions for the function F_n, for $n = 2, 3, 4, 6, 3/2, 4/3$ and $6/5$ are given in ref. [1] as well as the general results

$$F_n(\theta) = \theta^{(2n-1)/n}\sum_{s=1}^{\infty}\frac{n(-\theta)^{(s-1)}}{(ns-1)(ns+n-1)} \quad \text{for} \quad \theta \leqslant 1 \quad (9)$$

and

$$F_n(\theta) = \frac{(1+\theta)\pi/n}{\sin(\pi/n)} + \theta^{1-1/n} \times \sum_{s=0}^{\infty}\frac{n(-\theta)^{-s}}{(ns+1)(ns-n+1)} \quad \text{for} \quad \theta \geqslant 1. \quad (10)$$

The function I_n is related to F_n by

$$I_n(\theta^{1/n}) = \frac{F_n(\theta) + \theta^{(1-1/n)}/(n-1)}{(1+\theta)}. \quad (11)$$

In ref. [1], it was suggested that the bracketed term in equation (8) is negligible so that

$$q = \theta_0^{1/n}F_n(\theta). \quad (12)$$

PRELIMINARY COMPARISONS FOR DROPWISE CONDENSATION OF MERCURY

As seen in ref. [4], equation (12) gave fair agreement with experimental values of the surface heat-transfer coefficient for condensation of mercury [4, 6–8]. This is illustrated in Fig. 1. However, as may be seen from Fig. 2, the experimental Q–ΔT relationship showed clear evidence of curvature (i.e. the heat-transfer coefficient is not independent of ΔT) and the values of α given in Fig. 1 are mean values for the respective experimental ranges. Figure 1 indicates that the theoretical values of α are virtually independent of ΔT.

It may be seen in Fig. 2 that, while there are significant discrepancies between the two data sets [4, 7], both show curvature of the Q–ΔT relationship of opposite sign to that found for steam and ethanediol [2, 3]. In the case of ref. [6] detailed Q–ΔT results are not available while the data reported in ref. [8] are considered less reliable than those of ref. [4] as discussed in the latter paper. The data of ref. [4] are used here for more detailed comparisons with theory.

K_1, K_2, K_3 and n should all be of order unity and the values used for the theoretical curve in Fig. 1 are those appropriate for steam. An extensive curve fitting exercise using K_1, K_2, K_3 and n as adjustable parameters, carried out by the present authors, indicated that K_1 had to be much lower than

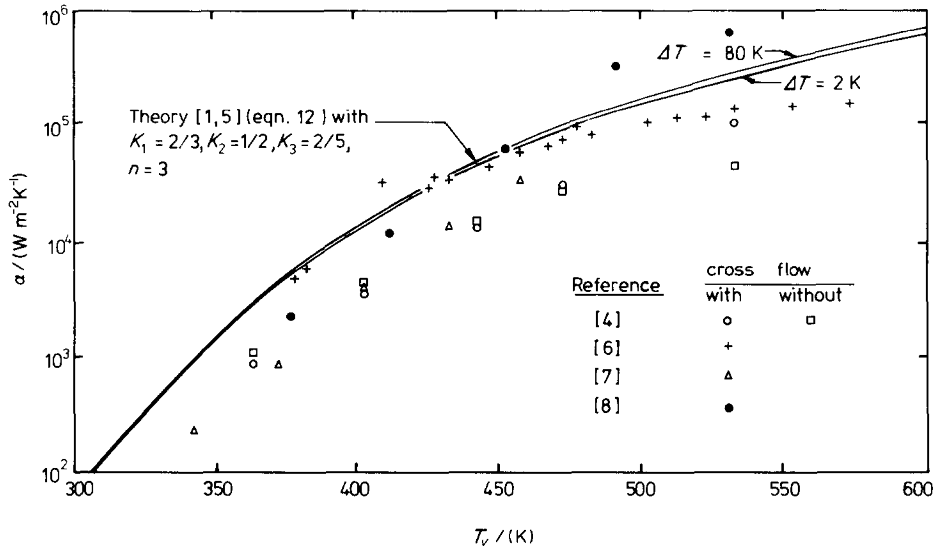

FIG. 1. Comparison between theory and experiment on the basis of mean heat-transfer coefficients.

anticipated* and K_2 had to be significantly higher than anticipated, to give an improved fit for the mercury data. The theoretical Q–ΔT lines always had very slight curvature (almost constant α) of opposite sign to that of the data, i.e. adjustment of the constants, even outside their estimated ranges, did not produce the same trend as that shown by the data.

CONSIDERATION OF THE APPROXIMATIONS IN THE ORIGINAL THEORY [1]

Approximation of equation (8) *by equation* (12)

This approximation is equivalent to setting the upper limit of the integral in equation (7) to infinity so that equation (12) overestimates the heat flux. The error is small when the larger drops contribute little to the heat-transfer rate. This is true for the case of steam (under consideration in ref. [1]) where the use of equation (12) rather than equation (8) gives a maximum error of between 1 and 2% for the ranges of vapour temperature and ΔT obtained in the experimental investigations. For mercury, however, with a substantially higher thermal conductivity than that of water, it is not evident that this approximation is valid. In the present work, calculations for mercury on the basis of equation (8) show that, at low pressure, equation (12) overestimates the heat flux by a factor of around 2. A further extensive curve-fitting study, using equation (8) rather than equation (12) was carried out. However, this still resulted in almost straight theoretical Q–ΔT lines, but with curvature of opposite sign to the data, and unsatisfactory values of K_1 and K_2 (see Fig. 3).

Linear approximation to the saturation curve

A second approximation in the theory of dropwise condensation [1] is the use of a linear approximation, equation (6), to the saturation curve, to estimate the interface temperature drop $T_v - T_s$. As illustrated in Fig. 4, this may give a significant underestimate of the interface temperature drop when $P_v - P_s$ is large, as is the case for mercury at low pressure. As indicated earlier, if this approximation is not adopted it is necessary to resort to numerical integration to determine Q. In the present work equation (1) was evaluated numerically for the experimental conditions used in ref. [4]. For given values of T_v and ΔT, equations (2) and (3) were solved by iteration to give Q_b, for each value of r. In the first instance the provisional values

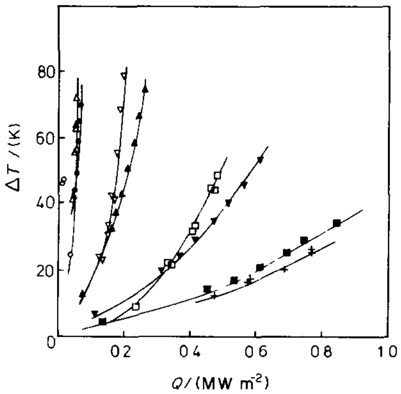

FIG. 2. Experimental data illustrating curvature of Q–ΔT relationship. Reference [4], data obtained with no net vapour downflow, values of T_v: ●, 363 K; ▲, 403 K; ▼, 443 K; ■, 473 K; data for highest temperature, 573 K, omitted—effects of non-condensing gas suspected (see ref. [9]). Reference [7] values of T_v: ○, 343 K; △, 373 K; ▽, 403 K; □, 433 K; +, 458 K.

---

* There are grounds to expect that K_1 should be larger for mercury than for steam [5].

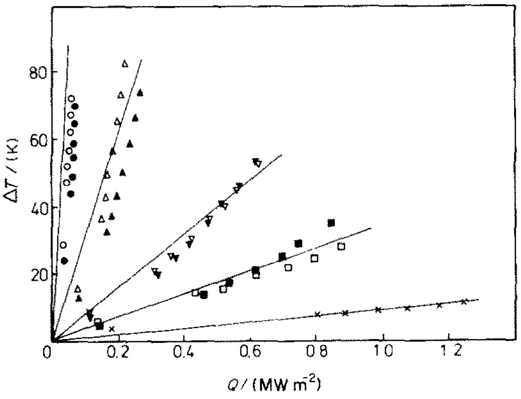

FIG. 3. Comparison of experimental data [4] with theory on the basis of equation (8) with $n = 3, K_1 = 0.052, K_2 = 3.2, K_3 = 0.5$, ref. [4] values of T_v: ●, ○, 363 K; ▲, △, 403 K; ▼, ▽, 443 K; ■, □, 473 K; ×, 573 K. The first symbol (filled) in each case denotes no net vapour downflow while the second symbol (open) and × denote presence of net vapour downflow.

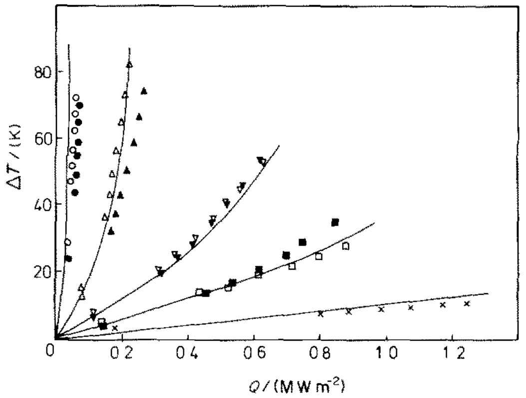

FIG. 5. Comparison of experimental data [4] with theoretical results obtained without using linear approximation to the saturation curve. For the theory (lines) $n = 3.7$, $K_1 = 2.22$, $K_2 = 0.94$, $K_3 = 1.15$. The data symbols are defined in Fig. 3.

$n = 3$, $K_1 = K_2 = 1$ and $K_3 = 0.5$ were used. The Q–ΔT curves were found to be in better agreement with the experimental data than when these constants were used in the original version of the theory. More importantly the new theoretical results exhibited curvature of the same sign as that of the data. Consideration was given to determination of the values of the four constants to give best agreement with the experimental data. The most straightforward procedure is to seek to minimize the sum of squares of residuals (difference between calculated and experimental values) of Q. A set of trial values of the constants was adopted and, for each experimental value of T_v and ΔT, the abovementioned numerical integration (with iteration at each value of r to solve equations (2) and (3) for Q_b) was carried out and the residual obtained. This was done for all data points and new values of the constants selected so as to reduce the sum of squares of residuals of Q. In view of the large computing time requirement and of the fact that significant differences exist between the data of the different investigators, a full minimization was not carried out. Sufficient runs were, however, conducted to satisfy the authors that good agreement could be obtained when the constants took more reasonable values than those required by the original theory. An improved fit is shown in Fig. 5. Better fits could have been found. The resulting constants would be somewhat different to those given in Fig. 5. Similarly, different (but reasonable) values would have been found by fitting the other data sets. We are not concerned here to decide which data set is most reliable or which are the 'best' constants for mercury. Rather we wish to point out that unsatisfactory aspects of previous comparisons between theory and measurements do not reflect flaws in the basic theoretical model but are due to invalid (for mercury) approximations used in the analysis.

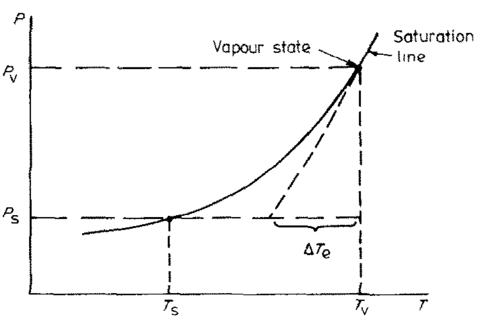

FIG. 4. Actual and estimated [1, 5] values of $T_v - T_s$.

CONCLUSION

Significant discrepancies between theory and experiment for dropwise condensation of mercury result from mathematical approximations used in ref. [1]. When these are not adopted, theoretical predictions are in better accord with experimental data but it is necessary to resort to numerical integration to determine the heat flux–temperature difference relationship.

Acknowledgement—The authors are grateful to Dr M. R. Nightingale for computing assistance.

REFERENCES

1. E. J. Le Fevre and J. W. Rose, A theory of heat transfer by dropwise condensation, *Proc. 3rd Int. Heat Transfer Conf.*, Chicago, Vol. 2, p. 362 (1966).
2. J. W. Rose, Further aspects of dropwise condensation theory, *Int. J. Heat Mass Transfer* **19**, 1363 (1976)
3. S. A. Stylianou and J. W. Rose, Dropwise condensation of ethanediol, *Physico-Chemical Hydrodynamics* **3**, 199 (1982).
4. J. Niknejad and J. W. Rose, Dropwise condensation of

mercury, *Proc. 6th Int. Heat Transfer Conf.*, Toronto, Vol. 2, paper CS-18, p. 483 (1978).

5. J. W. Rose, Dropwise condensation of mercury, *Int. J. Heat Mass Transfer* **15,** 1431 (1972).
6. M. N. Ivanovskii, V. I. Subbotin and Yu. V. Milovanov, Heat transfer with dropwise condensation of mercury vapour, *Teploenergetika* **14,** 81 (1967).
7. M. Kollera and U. Grigull, Untersuchung der Kondensation von Quecksilberdampf, *Wärme- und Stoffübertragung* **4,** 244 (1971).
8. S. Necmi and J. W. Rose, Heat transfer measurements during dropwise condensation of mercury. *Int. J. Heat Mass Transfer* **20,** 877 (1977).
9. J. Niknejad, An investigation of heat transfer during filmwise and dropwise condensation of mercury, Ph.D. thesis, University of London (1979).

DES COMPARAISONS ENTRE EXPERIENCE ET THEORIE POUR LA CONDENSATION EN GOUTTES DU MERCURE

Résumé—On montre que des approximations utilisées dans la théorie de la condensation en gouttes, pour évaluer l'intégrale donnant le flux thermique, conduisent à des erreurs significatives dans le cas du mercure. Quand ces approximations sont évitées, et l'intégrale calculée numériquement, on trouve un accord amélioré entre théorie et expérience.

VERGLEICH VON EXPERIMENT UND THEORIE BEI DER TROPFENKONDENSATION VON QUECKSILBER

Zusammenfassung—Es zeigt sich, daß Näherungen, wie sie in der Theorie der Tropfenkondensation bei der Berechnung des Integrals für die Wärmestromdichte verwendet werden, im Falle von Quecksilber zu wesentlichen Fehlern führen. Werden diese Näherungen vermieden und die Integration numerisch durchgeführt, so ist die Übereinstimmung von Theorie und Experiment deutlich besser.

СРАВНЕНИЕ МЕЖДУ ЭКСПЕРИМЕНТОМ И ТЕОРИЕЙ ДЛЯ КАПЕЛЬНОЙ КОНДЕНСАЦИИ РТУТИ

Аннотация—Показано, что использование в теории капельной конденсации удобных аппроксимаций для вычисления интеграла, по которому определяется величина теплового потока, приводит в случае ртути к значительным погрешностям. Если же не применять аппроксимаций, а находить интеграл численно, то согласие между теорией и экспериментом заметно улучшается.